GCSE to New A Level Transition in Mathematics

There are several topics that you covered in GCSE that also form part of A Level. While you understood them in May, you might be a little rusty by the time September comes around. These are fundamental skills that form the foundation for the rest of the A Level course. While some time will be devoted to these topics in the first half-term, time pressures prevent us from teaching them from scratch.

So, in order to hit the ground running in September, you are expected to complete the questions below.

At the end of August, you will be e-mailed the mark-schemes for these questions (they should be available here: https://www.mathspanda.com/ASMa.html). Therefore, on the first school day in September, you are expected to bring full written and marked solutions. In addition, please print out and fill in the final page of this document.

COORDINATE GEOMETRY

For the points $A(x_1, y_1)$ and $B(x_2, y_2)$:

- **Mid-point**: $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$
- **Distance between two points**: AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Gradient between two points**: Gradient of the line AB = $\frac{y_2-y_1}{x_2-x_1}$
- **Equation of the line with gradient m and passing through point A**: $y - y_1 = m(x - x_1)$
- **Parallel lines** – have the same gradient.
- **Perpendicular lines** – the product of perpendicular gradients is -1. To find a perpendicular gradient, flip the gradient and change the sign i.e. the gradient 2/3 is perpendicular to -3/2.

1.

A is the point $(2, 7)$ and B is the point $(-1, -2)$.

(i) Find the equation of the line through A parallel to the line $y = 4x - 5$, giving your answer in the form $y = mx + c$. [3]

(ii) Calculate the length of AB, giving your answer in simplified surd form. [3]

(iii) Find the equation of the line which passes through the mid-point of AB and which is perpendicular to AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

2.

(i) Find the gradient of the line l which has equation $x + 2y = 4$. [1]

(ii) Find the equation of the line parallel to l which passes through the point $(6, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [3]

(iii) Solve the simultaneous equations

$$y = x^2 + x + 1 \quad \text{and} \quad x + 2y = 4. \qquad [4]$$

3.

A is the point $(4, -3)$ and B is the point $(-1, 9)$.

(i) Calculate the length of AB. [2]

(ii) Find the coordinates of the mid-point of AB. [2]

(iii) Find the equation of the line through $(1, 3)$ which is parallel to AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

ALGEBRA

Expanding brackets

4.

Simplify $(2x + 5)^2 - (x - 3)^2$, giving your answer in the form $ax^2 + bx + c$. [3]

5.

(i) Simplify $(3x+1)^2 - 2(2x-3)^2$. [3]

(ii) Find the coefficient of x^3 in the expansion of

$$(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).$$ [2]

6.

(i) Expand and simplify $(2x+1)(x-3)(x+4)$. [3]

(ii) Find the coefficient of x^4 in the expansion of

$$x(x^2 + 2x + 3)(x^2 + 7x - 2).$$ [2]

QUADRATICS

- **Quadratic formula** for the equation $ax^2 + bx + c = 0$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Completing the square form**: $y = p(x+q)^2 + r$
- **Simultaneous equations (linear/non-linear)**

7.

Solve the simultaneous equations

$$4x^2 + y^2 = 10, \qquad 2x - y = 4.$$ [6]

Completing the square

8.

Express $x^2 - 12x + 1$ in the form $(x-p)^2 + q$. [3]

9.

Express $x^2 - 5x + \frac{1}{4}$ in the form $(x-a)^2 - b$. [3]

10.

Given that $3x^2 + bx + 10 = a(x+3)^2 + c$ for all values of x, find the values of the constants a, b and c. [4]

Inequalities

11.

Solve the inequalities

(i) $3(x-5) \leqslant 24$, [2]

(ii) $5x^2 - 2 > 78$. [3]

12.

Solve the inequalities

(i) $-35 < 6x + 7 < 1$, [3]

(ii) $3x^2 > 48$. [3]

TRANSFORMATIONS OF GRAPHS

Function	Transformation	Effect on coordinates
y = f(x) + k	Vertical translation, k units up	Add k to the y-coordinate
y = f(x + k)	Horizontal translation, k units left	Subtract k from the x-coordinate
y = -f(x)	Reflection in the x-axis	Change the sign of the y-coordinates
y = f(-x)	Reflection in the y-axis	Change the sign of the x-coordinates

Transformations of graphs

13.

(i) The curve $y = x^2$ is translated 2 units in the positive x-direction. Find the equation of the curve after it has been translated. [2]

(ii) The curve $y = x^3 - 4$ is reflected in the x-axis. Find the equation of the curve after it has been reflected. [1]

14.

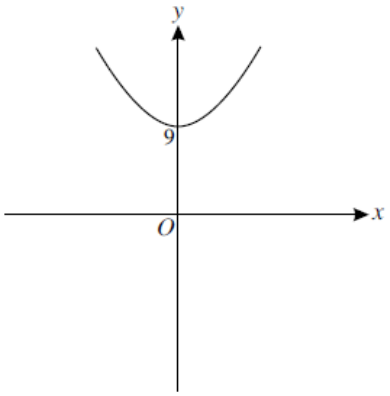


Fig. 1

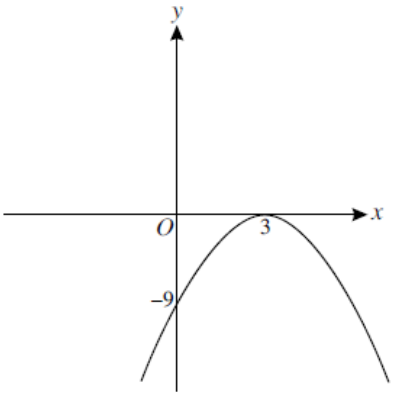


Fig. 2

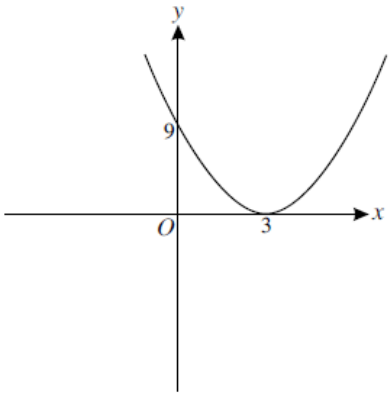


Fig. 3

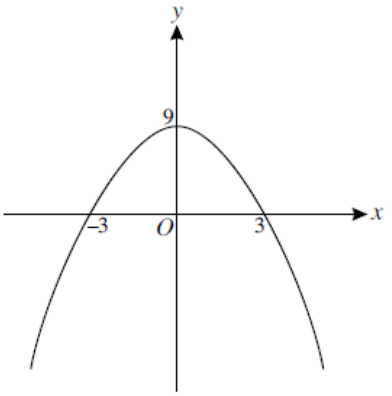


Fig. 4

(i) Each diagram shows a quadratic curve. State which diagram corresponds to the curve

(a) $y = (3 - x)^2$, [1]

(b) $y = x^2 + 9$, [1]

(c) $y = (3 - x)(x + 3)$. [1]

(ii) Give the equation of the curve which does not correspond to any of the equations in part **(i)**. [2]

INDICES AND SURDS

Laws of indices

- $x^a \times x^b = x^{a+b}$ i.e. when multiplying, we add the powers.
- $x^a \div x^b = x^{a-b}$ i.e. when dividing, we subtract the powers.
- $(x^a)^b = x^{ab}$ i.e. when raised to the power, we multiply the powers.
- Zero index $x^0 = 1$
- Negative indices $x^{-n} = \frac{1}{x^n}$ $\frac{1}{x^{-n}} = x^n$ $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$
- Fractional indices $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ $x^{-\frac{a}{b}} = \frac{1}{x^{\frac{a}{b}}}$

Surds

- Simplifying – find a square number that is a factor e.g. $\sqrt{28} = \sqrt{4 \times 7} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$
- Rationalising the denominator – multiplying top and bottom by the square root part of the denominator

  e.g. $\frac{20}{\sqrt{2}} = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2}$

15.

Solve the equations

(i) $x^{\frac{1}{3}} = 2$, [1]

(ii) $10^t = 1$, [1]

(iii) $\left(y^{-2}\right)^2 = \frac{1}{81}$. [2]

16.

Evaluate

(i) 6^0, [1]

(ii) $2^{-1} \times 32^{\frac{4}{5}}$. [3]

17.

Simplify the following, expressing each answer in the form $a\sqrt{5}$.

(i) $3\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [3]

18.

Solve the equations

(i) $10^p = 0.1$, [1]

(ii) $\left(25k^2\right)^{\frac{1}{2}} = 15$, [3]

(iii) $t^{-\frac{1}{3}} = \frac{1}{2}$. [2]

19.

Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

20.

Express each of the following in the form $k\sqrt{2}$, where k is an integer:

(i) $\sqrt{200}$, [1]

(ii) $\frac{12}{\sqrt{2}}$, [1]

(iii) $5\sqrt{8} - 3\sqrt{2}$. [2]

TRIGONOMETRY

	Finding a side	Finding an angle
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B}$ Need: an angle and its opposite side plus another angle or side	$\frac{\sin A}{a} = \frac{\sin B}{b}$ Need: an angle and its opposite side plus another angle or side
Cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$ Need: 2 sides and the angle between them	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ Needed: all 3 sides
Area of a triangle	Area of a triangle = $\frac{1}{2}ab\sin C$ Need: 2 sides and the angle between them (like cosine rule – finding a side)	

Sine/cosine rule and area of a triangle

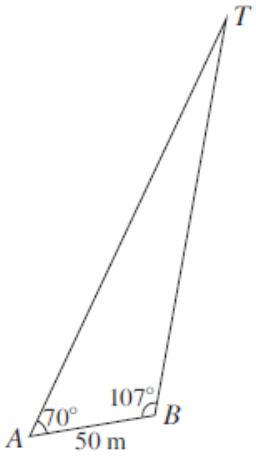
21.

The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

22.



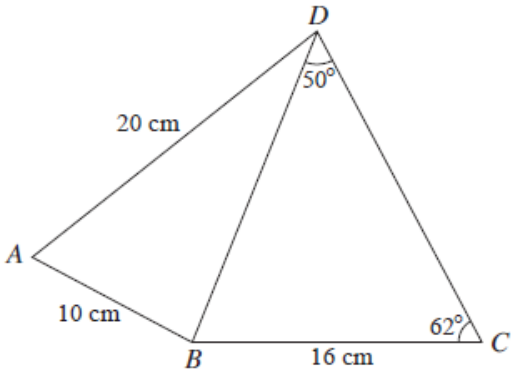
Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from a point A and then walk for 50 m in a straight line before taking another bearing from a point B. They find that angle TAB is 70° and angle TBA is 107° (see diagram).

(i) Find the distance of the tower from A. **[2]**

(ii) They continue walking in the same direction for another 100 m to a point C, so that AC is 150 m. What is the distance of the tower from C? **[3]**

(iii) Find the shortest distance of the walkers from the tower as they walk from A to C. **[2]**

23.



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

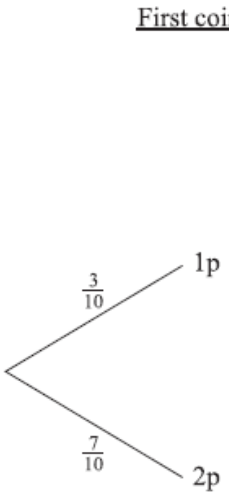
PROBABILITY

Probability

24.

A bag contains three 1p coins and seven 2p coins. Coins are removed at random one at a time, **without** replacement, until the total value of the coins removed is **at least** 3p. Then no more coins are removed.

(i) Copy and complete the probability tree diagram. [5]



Find the probability that

(ii) exactly two coins are removed, [3]

(iii) the total value of the coins removed is 4p. [3]

25.

(i) A biased coin is thrown twice. The probability that it shows heads both times is 0.04. Find the probability that it shows tails both times. [3]

(ii) Another coin is biased so that the probability that it shows heads on any throw is p. The probability that the coin shows heads exactly once in two throws is 0.42. Find the two possible values of p. [5]

26.

(i) A bag contains 12 red discs and 10 black discs. Two discs are removed at random, without replacement. Find the probability that both discs are red. [2]

(ii) Another bag contains 7 green discs and 8 blue discs. Three discs are removed at random, without replacement. Find the probability that exactly two of these discs are green. [3]

(iii) A third bag contains 45 discs, each of which is either yellow or brown. Two discs are removed at random, without replacement. The probability that both discs are yellow is $\frac{1}{15}$. Find the number of yellow discs which were in the bag at first. [4]

27.

A washing-up bowl contains 6 spoons, 5 forks and 3 knives. Three of these 14 items are removed at random, without replacement. Find the probability that

(i) all three items are of different kinds, **[3]**

(ii) all three items are of the same kind. **[3]**

STATISTICS

Statistics

28.

The diameters of 100 pebbles were measured. The measurements rounded to the nearest millimetre, x, are summarised in the table.

x	$10 \leqslant x \leqslant 19$	$20 \leqslant x \leqslant 24$	$25 \leqslant x \leqslant 29$	$30 \leqslant x \leqslant 49$
Number of stones	25	22	29	24

These data are to be presented on a statistical diagram.

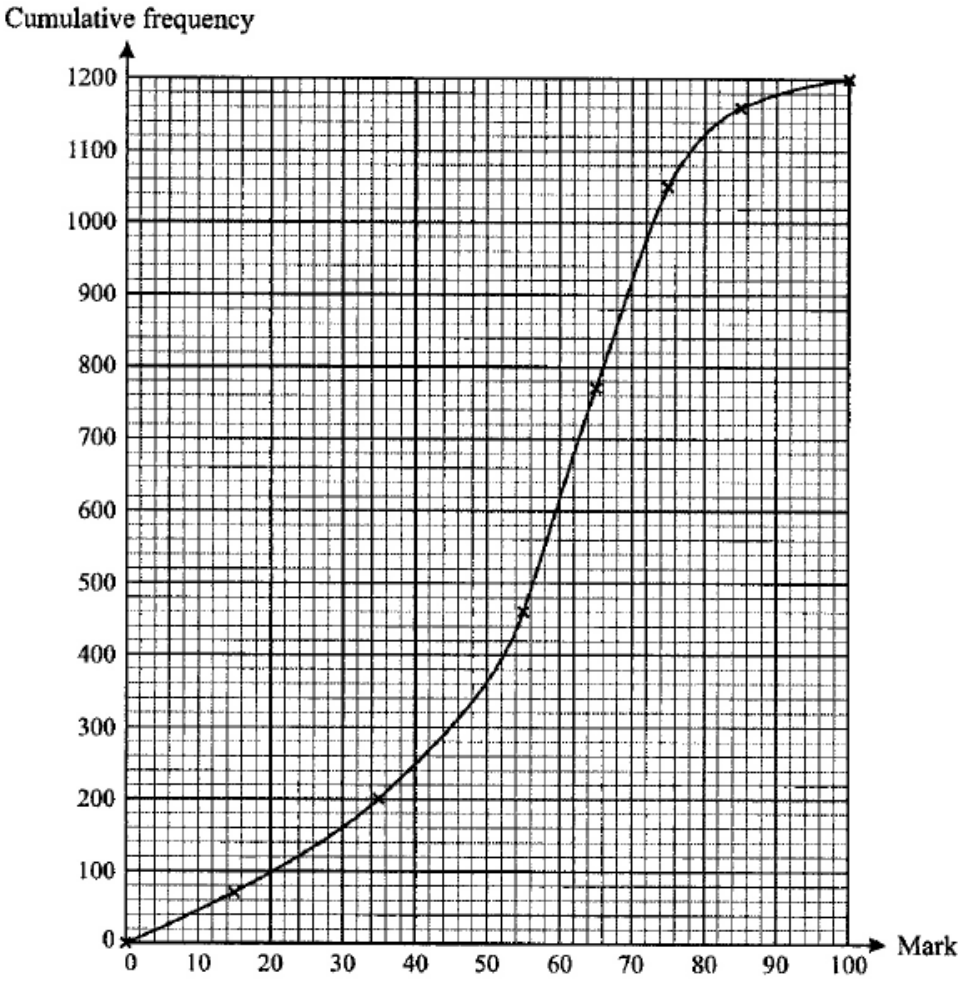
(i) For a histogram, find the frequency density of the $10 \leqslant x \leqslant 19$ class. **[2]**

(ii) For a cumulative frequency graph, state the coordinates of the first two points that should be plotted. **[2]**

(iii) Why is it not possible to draw an exact box-and-whisker plot to illustrate the data? **[1]**

29.

The examination marks obtained by 1200 candidates are illustrated on the cumulative frequency graph, where the data points are joined by a smooth curve.



Use the curve to estimate

(i) the interquartile range of the marks, [3]

(ii) x, if 40% of the candidates scored more than x marks, [3]

(iii) the number of candidates who scored more than 68 marks. [2]

Five of the candidates are selected at random, with replacement.

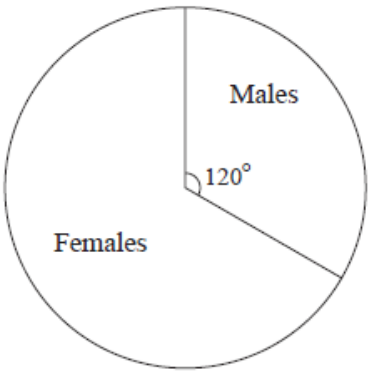
(iv) Estimate the probability that all five scored more than 68 marks. [3]

It is subsequently discovered that the candidates' marks in the range 35 to 55 were evenly distributed — that is, roughly equal numbers of candidates scored 35, 36, 37, …, 55.

(v) What does this information suggest about the estimate of the interquartile range found in part **(i)**? [2]

30.

(i) The numbers of males and females in Year 12 at a school are illustrated in the pie chart. The number of males in Year 12 is 128.

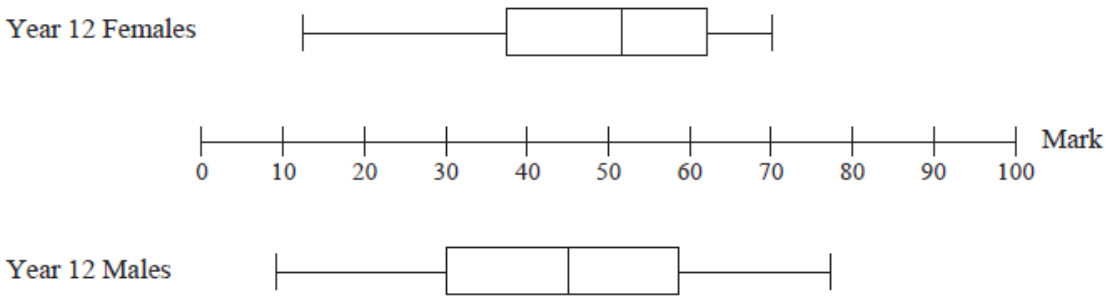


Year 12

(a) Find the number of females in Year 12. [1]

(b) On a corresponding pie chart for Year 13, the angle of the sector representing males is 150°. Explain why this does not necessarily mean that the number of males in Year 13 is more than 128. [1]

(ii) All the Year 12 students took a General Studies examination. The results are illustrated in the box-and-whisker plots.



(a) One student said "The Year 12 pie chart shows that there are more females than males, but the box-and-whisker plots show that there are more males than females."

Comment on this statement. [1]

(b) Give two comparisons between the overall performance of the females and the males in the General Studies examination. [2]

(c) Give one advantage and one disadvantage of using box-and-whisker plots rather than histograms to display the results. [2]

(iii) The mean mark for 102 of the male students was 51. The mean mark for the remaining 26 male students was 59. Calculate the mean mark for all 128 male students. [3]

Please print out and fill in this form, and then give it to your teacher in the first lesson

Name: **Teachers:**

GCSE grade(s):

Record your mark for each question here

Topic	Question	Out of	I got
Coordinate geometry	1	12	
Coordinate geometry	2	8	
Coordinate geometry	3	8	
Expanding brackets	4	3	
Expanding brackets	5	5	
Expanding brackets	6	5	
Simultaneous equations (linear/non-linear)	7	6	
Completing the square	8	3	
Completing the square	9	3	
Completing the square	10	4	
Inequalities	11	5	
Inequalities	12	6	
Transformations of graphs	13	3	
Transformations of graphs	14	5	
Indices and surds	15	4	
Indices and surds	16	4	
Indices and surds	17	5	
Indices and surds	18	6	
Indices and surds	19	4	
Indices and surds	20	4	
Sine/cosine rule and area of a triangle	21	5	
Sine/cosine rule and area of a triangle	22	7	
Sine/cosine rule and area of a triangle	23	5	
Probability	24	8	
Probability	25	9	
Probability	26	6	
Probability	27	11	
Statistics	28	13	
Statistics	29	10	
Statistics	30	5	
	Totals	**181**	

My percentage is: **%**

My grade is:

Grade boundaries						
Grade	**A***	**A**	**B**	**C**	**D**	**E**
Percentage	**85%**	**77%**	**68%**	**60%**	**52%**	**44%**

I need to work on/had difficulty with: